IB DIPLOMA PROGRAMME
PROGRAMME DU DIPLÔME DU BI
PROGRAMA DEL DIPLOMA DEL BI

FURTHER MATHEMATICS
STANDARD LEVEL
PAPER 2

Thursday 16 May 2002 (morning)

2 hours

INSTRUCTIONS TO CANDIDATES

- Do not open this examination paper until instructed to do so.
- Answer all the questions.
- Unless otherwise stated in the question, all numerical answers must be given exactly or to three significant figures.
- Write the make and model of your calculator on the front cover of your answer booklets *e.g.* Casio *fx-9750G*, Sharp EL-9600, Texas Instruments TI-85.

*Please start each question on a new page. You are advised to show all working, where possible. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Incorrect answers with no working will normally receive **no** marks.*

1. *[Maximum mark: 18]*

(i) A farmer produces duck eggs at his farm. He estimates that he will make a profit of 30 cents per egg on 20% of the eggs, 20 cents per egg on 50%, 10 cents per egg on 20%, no profit on 6% and a loss of 10 cents per egg on 4% of the eggs.

(a) Construct the probability distribution table and calculate the expected profit per egg. *[3 marks]*

(b) This year he has estimated that he will produce 700 000 eggs. What is the probability, correct to four decimal places, that the farmer will earn more than \$123 000 this year? *[6 marks]*

(ii) An international company owns two factories, one in Europe and one in Australia. The accidents in the factories in one month follow the Poisson distribution with parameters λ_1 and λ_2 respectively, and happen independently.

(a) Show that the probability that the company will have only one accident in the factories in a given month is given by the formula $p = (\lambda_1 + \lambda_2)\,\mathrm{e}^{-(\lambda_1+\lambda_2)}$. *[3 marks]*

(b) Derive the formula for k accidents in the factories (where $k \geq 0$) . What can you deduce from your formula about the distribution of the number of accidents? *[6 marks]*

2. *[Maximum mark: 22]*

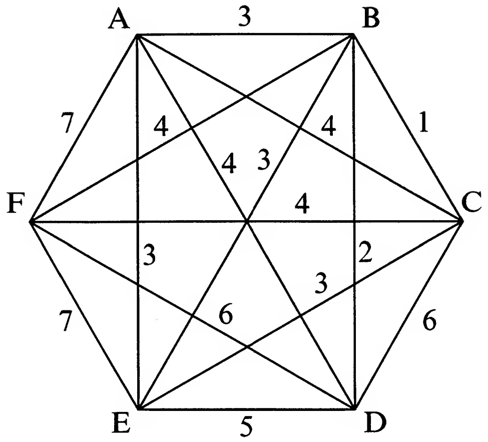
(i) The operation $\circ$ is defined on the set $S = \{x \mid -1 < x < 1,\ x \in \mathbb{R}\}$ by $x \circ y = \dfrac{x+y}{1+xy}$, $x, y \in S$.

Determine whether S forms an Abelian group under $\circ$, giving reasons. *[14 marks]*

(ii) The function $f : A \to B$ is defined by $f(x, y) = (x\cos y, \sin y)$, where $A = [0, \infty[\times \left[0, \dfrac{\pi}{2}\right[$ and $B = [0, \infty[\times [0, 1[$. Investigate whether f is injective, surjective or both. If possible, define the inverse function. If not, explain why not. *[8 marks]*

3. *[Maximum mark: 20]*

(i) The following diagram represents a graph G.



Graph G

(a) Use an appropriate algorithm to find the minimum spanning tree of G and state its weight. *[4 marks]*

(b) Adapt Prim's algorithm to find the maximum weighted tree and state its weight. Explain how the adapted algorithm works. *[5 marks]*

(ii) (a) The sum of the digits of a three-digit number of the form *abb* is divisible by 7. Show that the number itself is divisible by 7. *[4 marks]*

(b) Use Euclid's algorithm to find the smallest positive integers x and y that satisfy the equation $57x - 13y = 7$. *[7 marks]*

4. *[Maximum mark: 20]*

(i) (a) Find the error in approximating the integral $\int_0^2 xe^{-2x}dx$ by using Simpson's rule with 10 strips. *[3 marks]*

(b) How many strips are necessary so that the error term is less than 5×10^{-6}? *[3 marks]*

(ii) (a) Find the integral $\int \frac{dx}{x(x-1)(x-2)}$. *[7 marks]*

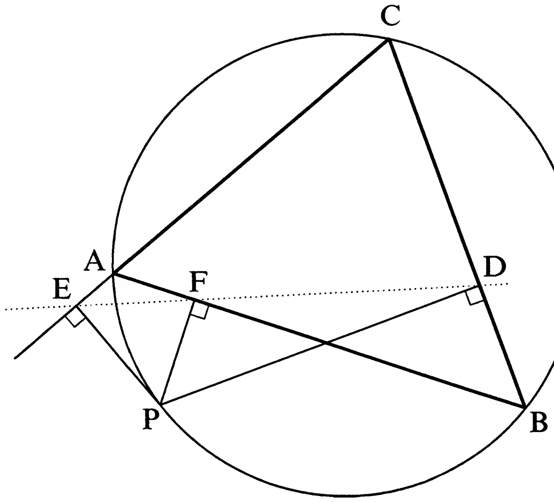
(b) **Hence** determine whether the following series converges.

$$\frac{1}{2\times3\times4}+\frac{1}{3\times4\times5}+\frac{1}{4\times5\times6}+\ldots+\frac{1}{(n-2)\times(n-1)\times n}+\ldots,\ n \geq 4\ .$$ *[7 marks]*

5. *[Maximum mark: 20]*

(i) (a) State Menelaus' theorem and its converse. *[3 marks]*

(b) The following diagram shows a triangle ABC and its circumscribed circle. The point P is on the circle such that its perpendicular projections to the lines (BC), (CA) and (AB) are D, E and F respectively.



Show that the points D, E and F are collinear. *[7 marks]*

(ii) The parabola $y^2 = 4px$ is given.

(a) Prove that the tangent to the parabola at the point (x_0, y_0) has equation $yy_0 = 2p(x + x_0)$. *[4 marks]*

(b) The point D is on the parabola, and is not the vertex. The point L is the foot of the perpendicular from the point D to the directrix. Prove that the tangent to the parabola at D is the angle bisector of $\widehat{FDL}$, where F is the focus of the parabola. *[6 marks]*